Deductive Evaluation: Formal Code Analysis with Low User Burden

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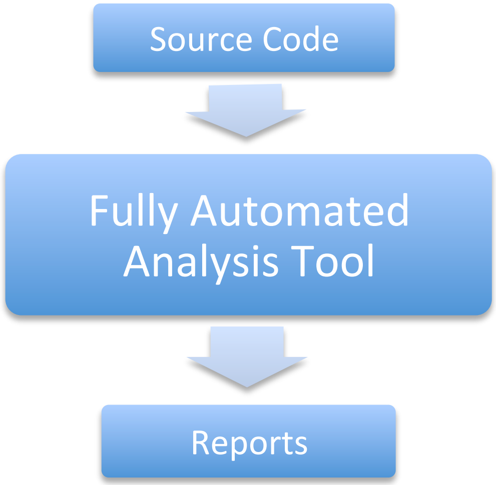
Landscape

- Formal code verification is enjoying a resurgence
  - Improved deduction (SMT solvers, primarily)
  - Recent tools: Frama-C, VCC, SPARK Pro (Ada)
- BUT:
  - Industry strongly prefers push-button methods
  - Code verifiers require effort
  - Will software engineers use them?



- Meanwhile, static analysis is fully automated
  - Many software developers have embraced them
  - But they only check well-formedness

Opportunities

- Can we automatically deduce functionality?
  - Yes! Discover, derive, infer code's execution behavior
  - Forgo traditional verification results
  - Challenge: Iteration is hard
- Our method analyzes code having loops
  - Adaptation of classical Floyd-Hoare verification methods
  - Loop invariant synthesis using iteration schemes
  - Annotation-free deductive evaluation of C functions
  - More complete form of symbolic evaluation/execution
  - Mechanized using PVS (Prototype Verification System)
  - Best-effort analysis; no guarantee of coverage

Opportunities (cont'd)

- Data-driven approach relies on a division of labor
  - Human assistance to create iteration scheme library
  - Full automation when applying them during evaluation
- Ease of use is a major goal
  - Encourages uptake by software engineers
  - Provides rigorous feedback on user's code
  - Augments existing tools and practices
- Filling a gap, finding a niche:

SA AI DE MC PV TP Custom

Lightweight Heavyweight

Example of Deductive Evaluation

C function:

```
int add_mult(
    unsigned int m,
    int n)
{
  int p = 0;
  unsigned int i = 0;
  while (i < m) {
    p += n;
    i++;
  }
  return p;
}
```

Evaluation result (PVS):

```
add_mult_deval
 [(IMPORTING
  iter_schemes@prog_types)
  m_0_: nat,
  n_0_: int] : THEORY
BEGIN
  . . .
  final: return_values =
    (# result_ :=
         m_0_ * n_0_ #)
  WFO: boolean = TRUE
END add_mult_deval
```

Example (cont'd)

```
IMPORTING iter_schemes@top

p_0_: int = 0
i_0_: nat = 0
result_0_: int
return_values: TYPE =
  [# result_: int #]

% Analyzing while loop at depth 1.
% Found dynamic variables: p, i
% Found static variables: m, n
% Found possible index variables: i

% Values at top of loop:
k_1_: nat  % implicit loop index
p_1_: int  % dynamic variable
i_1_: nat  % dynamic variable

% Effects of loop body:
p_2_: int = p_1_ + n_0_
i_2_: nat = i_1_ + 1
```

```
% Invariants for loop index i
% (scheme loop_index_recur):
%   (index_var_expr . i_1_ = k_1_)
%   (iter_k_expr . k_1_ = i_1_)
%   (initial_bound . TRUE)
%   (final_bound . i_1_ < 1 + m_0_)

% Invariants for variable p
% (scheme arith_series_recur):
%   p_1_ = (k_1_ * n_0_)

% Values of dynamic variables on
% (normal) loop exit:
k_2_: nat = m_0_
i_3_: nat = m_0_
p_3_: int = m_0_ * n_0_

% End of for/while loop at depth 1.
```

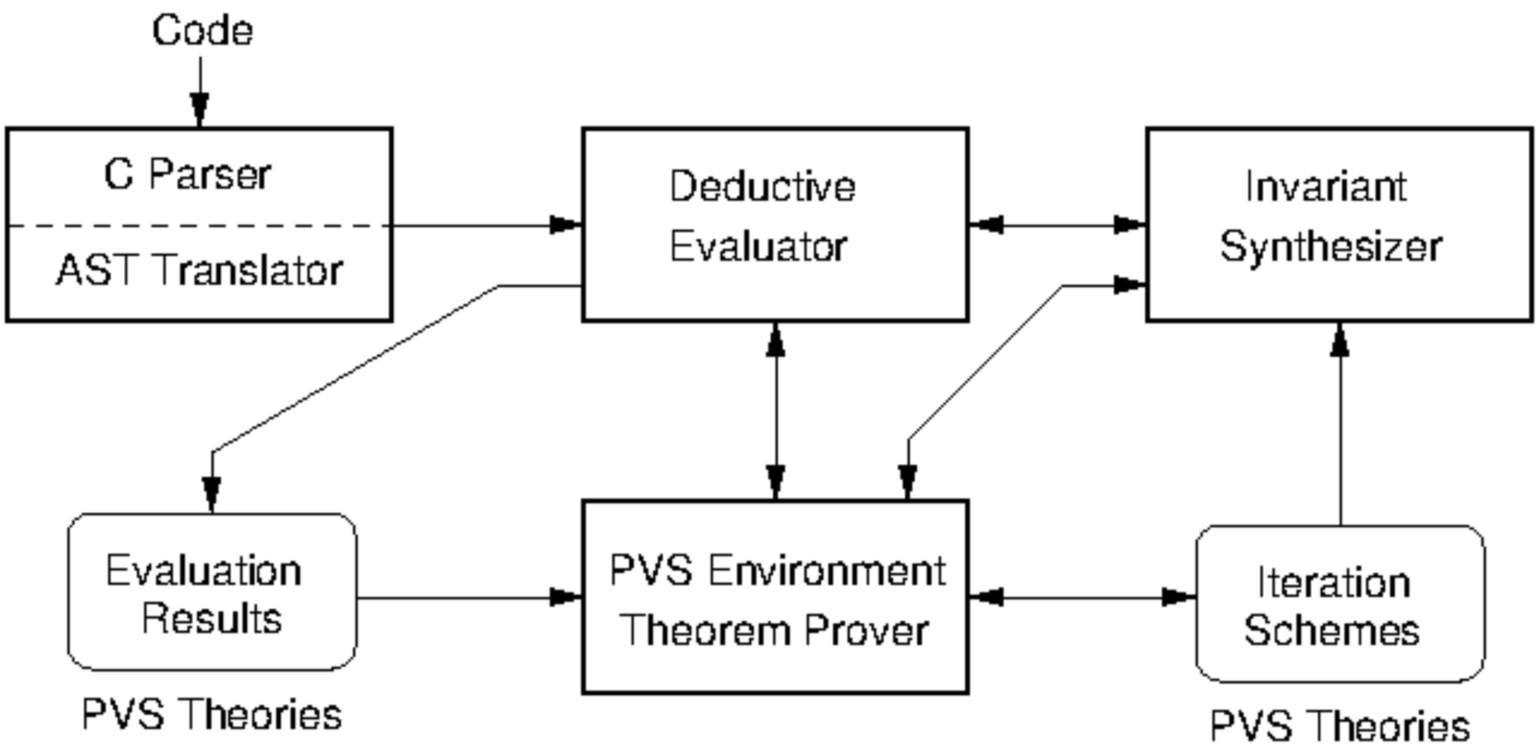
Features of PVS

- PVS (by SRI International) is both a language and a suite of deduction tools
  - Classical higher order logic with typing
  - Powerful interactive theorem prover
  - Prover also can be invoked programmatically
  - Tools hosted within the Emacs editor
- Relevant language features
  - Declarations grouped into parameterized theories
  - Predicate subtypes are crucial: { x : T | P(x) }
  - Function types are versatile; used to model arrays: [below(n) -> int]
- Uninterpreted constants model program values
  - Example: n_1_: {n: int | 0 <= n AND n < q}

C Features Supported

- Current fragment of C is modest
  - Types int, unsigned int and arrays of int
  - Function declarations and most statements
  - Function parameter mechanism
- Limitations and unsupported features
  - Integer types are unbounded
  - No side effects in expressions
  - No parameter aliasing (e.g., overlapping arrays)
  - No pointers (yet)
  - No declarations other than functions

Prototype Tool Chain



Evaluator, Synthesizer: Common Lisp
AST Translator: Python
C Parser: Open-source tool (Python)
Emacs Interface: Emacs Lisp

Invariant Concepts

- Non-iterative code segments can be analyzed via:
  - Predicate transformation
  - Proof rules from a program logic (e.g., Hoare logic)
  - Symbolic evaluation/execution
- Invariants are needed to capture loop behavior
  - In verification tools, normally provided by users
  - Generally considered a tedious, error-prone activity
- Typical proof rule for while-loop:
  - Given: $P \rightarrow Q \wedge \{B \wedge Q\}\ S\ \{Q\} \wedge Q \rightarrow (R \vee B)$
  - Infer: $\{P\}$ while B do S $\{R\}$
- Derivation of invariants is undecidable in general
  - Use tractable domains, heuristics or predefined schemes

Analysis Approach

- Invariant synthesis based on recurrence relations
  - Generalized for predicates
  - Iteration schemes expressed as PVS theories
  - Templates and patterns derived from theories
  - Applied during analysis using matching and proving
- Deductive evaluation of C code
  - Based on Floyd-Hoare verification concepts
  - No verification conditions
  - Instead, perform on-the-fly analysis and proof
  - Predicate subtypes play a key role
  - Iteration schemes are searched, invariants are derived
  - Fully automatic, strongest-postcondition analysis

Predicate Recurrence Relations

- Schemes formalize generalized recurrence relations
  - Recurrence: I(u,0): u = 1; R(u,v,k): v = 2*u
  - Solution: P(u,k): u = 2^k
  - Prove: I(u,0) -> P(u,0); P(u,k) $\wedge$ R(u,v,k) -> P(v,k+1)
  - Enables solutions to be Boolean expressions
- PVS formulation uses structured predicate definition
  - Labeled conditions and solution components
  - Implicit loop index k used in every scheme
  - Optional declaration for auxiliary facts
  - Inductive proof that solution satisfies recurrence
  - Meta-model expressed in separate theories

Example Scheme 1

```
arith_series_recur : THEORY
  BEGIN
  dyn_vars:    TYPE = int
  stat_vars:   TYPE = int
  IMPORTING recur_pred_defn[dyn_vars, stat_vars]
  k:         VAR nat
  I,U,V:     VAR dyn_vars
  S,W:       VAR stat_vars
  recur_type: recurrence_type = var_function

  recurrence(I, S)(U, V, k): recur_cond = . . .
  solution(I, S)(U, k): invar_list =  . . .

  recur_satis: LEMMA sat_recur_rel(solution, recurrence)

  END arith_series_recur
```

Example Scheme 1 (cont'd)

```
arith_series_recur : THEORY
  . . .
  recurrence(I, S)(U, V, k): recur_cond =
      LET s0 = I, d = S, u = U, v = V IN
        (# each := (: (iter_effect, v = u + d) :),
           once := (: :)
         #)

  solution(I, S)(U, k): invar_list =
      LET s0 = I, d = S, u = U IN
        (: (func_val_expr, u = k * d + s0),
           (initial_bound,
            IF d < 0 THEN u <= s0 ELSE u >= s0 ENDIF)
         :)
  . . .
  END arith_series_recur
```

Example Scheme 2

```
loop_index_recur : THEORY
  . . .
  dyn_vars:   TYPE = int
  stat_vars:  TYPE = [nzint, int, real_rel]
  . . .
  recurrence(I, S)(U, V, k): recur_cond =
      LET i0 = I, (d, n, R) = S, i = U, v = V IN
        (# each := (: (iter_effect,  v = i + d),
                      (while_cond,   R(i, n)) :),
           once := (: (dyn_init,     R(i0, n + d)),
                      (stat_cond,
                       R = reals.< OR R = reals.>) :)
         #)
  . . .
  END loop_index_recur
```

Example Scheme 2 (cont'd)

```
solution(I, S)(U, k): invar_list =
    LET i0 = I, (d, n, R) = S, i = U IN
        (: (index_var_expr,
            i = id(LAMBDA (k: nat): k * d + i0)(k)),
           (iter_k_expr,
            k = id(LAMBDA (i: int): (i - i0) / d)(i)),
           (initial_bound,
            IF d < 0 THEN i <= i0 ELSE i0 <= i ENDIF),
           (final_bound,
            R(i0, n + d) IMPLIES R(i, n + d)) :)

facts(I, S)(U, k): aux_fact_list =
    LET i0 = I, (d, n, R) = S, i = U IN
        (: (final_index_value,
            R(0, d) AND NOT R(i, n) IMPLIES
              i = n + mod(i0 - n, d)),
           (final_k_value,
            R(0, d) AND NOT R(i, n) IMPLIES
              k = ceiling((n - i0) / d)) :)
```

Evaluator Operation

- Deductive evaluator accepts C in intermediate form
  - ASTs rendered as Lisp s-expressions
- Evaluator processes C statements within a function
  - Process is similar to symbolic execution
  - Handles extra paths due to {if, return, break} statements
  - PVS theory built incrementally during evaluation
  - PVS constants model C variables at change points
  - Predicate subtypes used to express constraints
- Loop handler finds invariants for dynamic variables
  - Iteration schemes searched
  - Matching applied to effects of loop body
  - Prover checks conditions and performs simplification
  - Final variable values at end of loop are derived
  - Schemes can depend on invariants found earlier

Evaluation Example 2

C function:

```
int add_mult_exp(
  unsigned int m, int n) {
  int p = 0;
  unsigned int d = m;
  int y = n;
  while (d > 0) {
    if (d % 2 == 1)
       p += y;
    y += y;
    d /= 2;
  }
  return p;
}
```

Evaluation result (PVS):

```
. . .

% Invariants for variable d
% (scheme div2_exp2_recur):
%   d_1_ =
%     floor((m_0_ / (2 ^ k_1_)))

% Invariants for variable y
% (scheme double_exp2_recur):
%   y_1_ = (n_0_ * (2 ^ k_1_))

% Invariants for variable p
% (scheme exp2_mult_recur):
%   p_1_ = m_0_ * n_0_ -
%     floor((m_0_ / (2 ^ k_1_)))
%       * (2 ^ k_1_) * n_0_

. . .
```

Array Handling

- Array indexing leads to well-formedness concerns
  - Ensure that index expressions are within bounds
  - Two declaration cases in C: (1) int A[N] and (2) int A[]
  - For (1), check that i < N (well-formedness condition, WFC)
  - For (2), add an implicit size parameter, then generate a well-formedness obligation (WFO) to ensure i < size
- Invariants help constrain array accesses within loops
  - When i < n for all iterations, can generate WFO: n <= size
  - Special schemes are provided to establish the bounds
  - WFOs must be enforced in the calling environment

Evaluation Example 3

C function:

```
void array_init(
        int A[],
        unsigned int n,
        int v)
{
  unsigned int i;
  for (i=0; i<n; i++)
    A[i] = v;
}
```

Evaluation result (PVS):

```
array_init_deval
 [(IMPORTING
  iter_schemes@prog_types)
  A_size_: posnat,
  A_0_:int_array(A_size_),
  n_0_: nat, v_0_: int ] : THEORY
BEGIN
  . . .
  val_A: {r_: int_array(A_size_) |
            FORALL (q: below(n_0_)):
              r_(q) = v_0_}
  final: return_values =
    (# A := val_A #)
  WFO: boolean = n_0_ <= A_size_
END array_init_deval
```

Conditional Loop Exits

- Loops can be exited via return and break statements
  - Give rise to additional exit paths
- In some contexts, loop exits can induce invariants
  - When exit condition is P, can often infer “not P” holds at the top of every iteration
  - One sufficient condition is that the loop index is the only dynamic variable P references
  - Allows us to conclude the following:
  - FORALL (j: below(k)): NOT P(j)
  - An iteration scheme is provided to handle this case

Evaluation Example 4

C function:

```
int linear_search(
      const int A[],
      unsigned int n,
      int v) {
  int i = 0;
  while (i < n) {
    if (A[i] == v)
      return i;
    i += 1;
  }
  return -1;
}
```

Evaluation result (PVS):

```
linear_search_deval
 [(IMPORTING iter_schemes@prog_types)
  A_size_: posnat,
  A_0_: int_array(A_size_),
  n_0_: nat, v_0_: int] : THEORY
BEGIN
 . . .
 val_result_: {r_: int |
    (((r_ = -(1)) AND
      (FORALL (j: below(n_0_)):
         NOT A_0_(j) = v_0_)) OR
     (A_0_(r_) = v_0_ AND
       (r_ < n_0_) AND (0 <= r_) AND
        (FORALL (j: below(r_)):
           NOT A_0_(j) = v_0_)))}
 final: return_values =
   (# result_ := val_result_ #)
 WFO: boolean = n_0_ <= A_size_
END linear_search_deval
```

Nested Loops

- Inner loop completed first
  - Outer loop evaluation encounters inner loop on main path within body
  - Inner loop is processed independently, resulting in derived effects
  - Those effects used to match a scheme for outer loop
  - Inferred invariants for outer loop reflect combined behavior

C function:

```
void bubble_sort(
    int A[],
    unsigned int nm1) {
  unsigned int i, j;
  int t;
  for (i=0; i<nm1; i++) {
    for (j=i+1; j<1+nm1;
         j++) {
      if (A[j] < A[i]) {
        t = A[i];
        A[i] = A[j];
        A[j] = t; }
      } } }
```

Evaluation Example 5

Evaluation result (PVS):

```
bubble_sort_deval
  [(IMPORTING iter_schemes@prog_types)
   A_size_: posnat,
   A_0_: int_array(A_size_),
   nm1_0_: nat] : THEORY
BEGIN
  . . .
  A_6_:
   {A: int_array(A_size_) |
       (FORALL
        (p: below((nm1_0_ - i_1_))):
         (A(i_1_) <= A(1 + p + i_1_)))
       AND permutation_of?(A, A_1_)
       AND
        (FORALL (p: below(A_size_)):
          ((p < i_1_) OR (nm1_0_ < p))
          IMPLIES A(p) = A_1_(p))}

  . . .

  val_A:
   {r_: int_array(A_size_) |
    ((FORALL (p: below(nm1_0_)):
       (r_(p) <= r_(1 + p))) AND
     permutation_of?(r_, A_0_))}

  final: return_values =
    (# A := val_A #)

  WFO: boolean =
       1 + nm1_0_ <= A_size_

END bubble_sort_deval
```

Inferring End-to-End Behavior

- Example: Lossless data compression

```
void data_comp(const int A[1000],
               unsigned int n, int C[1000]) {
  int B[1000];
  unsigned int m;
  m = compress(n, A, B);    /* B's format derived */
  decompress(m, B, C); }
```

- Try to evaluate decompress in context
- Two possible techniques:
  - Expand the function decompress in-line and evaluate
  - Set the type of formal parameter B in decompress to match constraint produced by evaluation of compress
- Expected inference is that C = A

Limitations

- Current prototype
  - Subset of C supported; no other languages yet
  - Small scale, slow performance
  - Matching is syntactic; canonical forms help
  - Too many TCCs (type correctness conditions) spawned
  - Need multi-pass evaluation for full treatment
  - NASA PVS libraries can help
- Overall method
  - Could support verification tools; not addressed yet
  - Synthesize PVS functions to mitigate code complexity
  - Need to populate iteration scheme library (> 1K ?)
  - Large scheme library is a design challenge for tools

Potential Uses, Outlook

- Usage possibilities
  - Development aid, symbolic debugging
  - Complement to unit testing
  - Reverse engineering of source code
  - Analyzer for component libraries, specialized software domains
  - Synthesis of invariants for verifiers and other tools
- Future outlook
  - Promising, but much work lies ahead
  - Could benefit from:
    - Tighter PVS integration
    - Data mining to help create iteration schemes
    - Use of SMT solvers and computer algebra systems
    - Integration with IDEs
  - Concepts should be portable to other theorem provers

Questions?

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